Their vs. There vs. They're

As with other homophones, words that sound alike, people often get confused with the terms their, there, and they're despite their very distinct uses and meanings. This is particularly evident in written English even among native speakers.

The word their is a possessive pronoun used to describe something "belonging to or associated with the people or things previously mentioned or easily identified."

"Texas has no business billing U.S. taxpayers for their own silly border patrols" Los Angeles Times

"Health Law Repeal Could Cost 18 Million Their Insurance, Study Finds"

New York Times

"In Middle East, Leaders Want Donald Trump to Be Their Friend" Wall Street Journal

Meanwhile, there is commonly used as an adverb meaning "in, at, or to that place or position."

"Samsung at CES: Ignore that burning Galaxy Note 7 over there" CNET

"Despite Inequality In South Africa, African Migrants Go There More Than To Europe"

AFKInsider

"Coachella's Most Surprising Performer Deserves To Be There"
Forbes

It may also be used as a pronoun "to introduce a sentence or clause in which the verb comes before its subject or has no complement."

"There is nothing wrong with members of Congress boycotting the inauguration" Los Angeles Times

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"Donald Trump on Unsubstantiated Dossier Allegations: 'There Is No Tape'"

ABC News

"There's a Surprise Leader in Canada's Race for Smart Global Jobs"
Bloomberg

On the other hand, the word they're is a contraction for "they are" with the apostrophe indicating that the letter "a" has been left out. It is often followed by an adjective or a verb ending in -ing.

"They're just a pair of PJs. Or are they? My case for the silly things that soothe the soul" Los Angeles Times

"Ten Signs They've Already Decided Who They're Going To Hire"
Forbes

"Why the Warriors' blowout win didn't prove that they're truly better than the Cavaliers" Fox Sports

In summary, if you're trying to indicate a direction or a location, you should use there while if you want to express possession, it should be their. On the other hand, they're is simply a contraction so you may try to substitute "they are" to see if you are using it correctly. Hopefully, these tips would help you avoid the confusion with these homophones.